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# Simulation approach for investigating dynamics of passenger matching problem in smart ridesharing system

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**Abstract**

Smart ridesharing system (SRS) is a ridesharing system integrated with information and communication technology. We formulate a passenger matching problem by considering user preference in SRS. This study aims to investigate day-to-day dynamics and characteristics of passenger matching problem in SRS. A modified stable roommates problem is adopted to represent this matching problem. The characteristics of day-to-day dynamics (e.g., existence, uniqueness, and stability of equilibria) of SRS are understood and revealed by a simulation of formulated model.

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*Keywords:* Smart ridesharing system; Dynamic ridesharing system; Day-to-day dynamics; Within-day dynamics; Passenger matching problem; Stable roommates problem

## 1. Introduction

Efficient utilization of existing transportation has been considered to mitigate traffic congestion and environmental problems, and reduce fuel consumption. In order to achieve these objectives, sharing goods or services has been proposed and practically executed, for example ridesharing service. Ridesharing is a sharing of a riding or driving vehicle to others who have similar travel itineraries. In recent years, ridesharing has emerged through smart devices by integrating with information and communication technology. This integration (known as smart ridesharing systems, SRS) has facilitated travelers to access ridesharing systems, service providers to efficiently operate ridesharing systems. Dynamic ridesharing system, which gathers travelers with similar itineraries to share rides on short-notice (Agatz et al., 2011), is also one of the SRS features. The general objective for ridesharing is to minimize total travel distance and/or time (Furuhata et al., 2013). Besides that, user preference and similarity have been considered in order to provide a more efficient travelers matching for ridesharing (Agatz et al., 2010). Kleiner et al. (2011) developed a matching model between driver and passenger by using parallel auctions.

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This model allows users to consider factors (e.g., reputation, social network status) other than travel distance and travel time. Besides that, Wang (2013) developed a model based on stable marriage problem to provide a stable matching between driver and passenger. Moreover, a matching between passengers based on a modified stable roommates problem has been developed by Thaithatkul et al. (2015a). These last two models perform the matching by considering user preference.

In this study, the ridesharing that we focus on is a sharing of a riding vehicle driven by a professional driver such as taxi-sharing systems. This kind of ridesharing has already been studied and developed the models and applications (Ma et al., 2015; Martinez et al., 2015; Uber, 2015); however, the study which considers passenger personal preference has been rarely seen. Therefore, we initially consider a one-on-one passenger matching problem by considering a user preference. The modified stable roommates problem (Thaithatkul et al., 2015a) is employed to represent this problem.

Dynamics of SRS system are described by mutual relationship between travelers' response (e.g., decision on participating in SRS) and travelers' experience (e.g., cost reduction, waiting time, ridesharing partner) of the system over time. The dynamics of SRS are essential to understand as they can describe the evolution, sustainability, long-term behavior of the system. For instance, how total travel distance is reduced when SRS is adopted with given OD matrix of participants can be described. Long term behavior of travel distance can be investigated using day-to-day dynamics model. Moreover, understanding characteristics of dynamics (e.g., existence, uniqueness, and stability of equilibria) under specific conditions is important for planning efficient transportation systems. If multiple equilibria exist; authority should be able to determine which equilibrium state is preferable in term of social cost, and take action to lead SRS to the preferable equilibrium state. A previous study (Thaithatkul et al., 2015b) investigated this problem under the simple condition which one-dimensional city was considered. Therefore, this study aims to investigate day-to-day dynamics of a formulated passenger matching problem in SRS under a more general condition which is two-dimensional city. As it is difficult to analytically investigate the dynamics of SRS under this condition, we thus investigate by simulating a formulated SRS model.

This paper is organized as follows. The proposed SRS is explained in section 2. The model formulation of this proposed SRS can be divided into three sub models which are static matching model (described in section 3), within-day model (described in section 4), and day-to-day model (described in section 5). The integration of these three models in our proposed SRS is described in section 6. We perform the simulation to investigate the day-to-day dynamics of formulated SRS model in section 7. The result of the simulation is also presented and discussed in this section. This paper is ended with the conclusion of our study.

## 2. Proposed Smart Ridesharing System

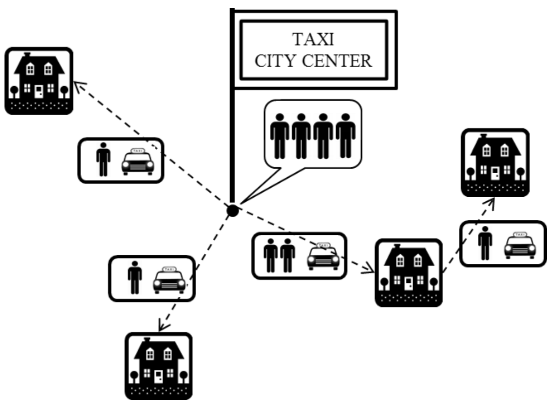


Fig. 1. Proposed ridesharing system.

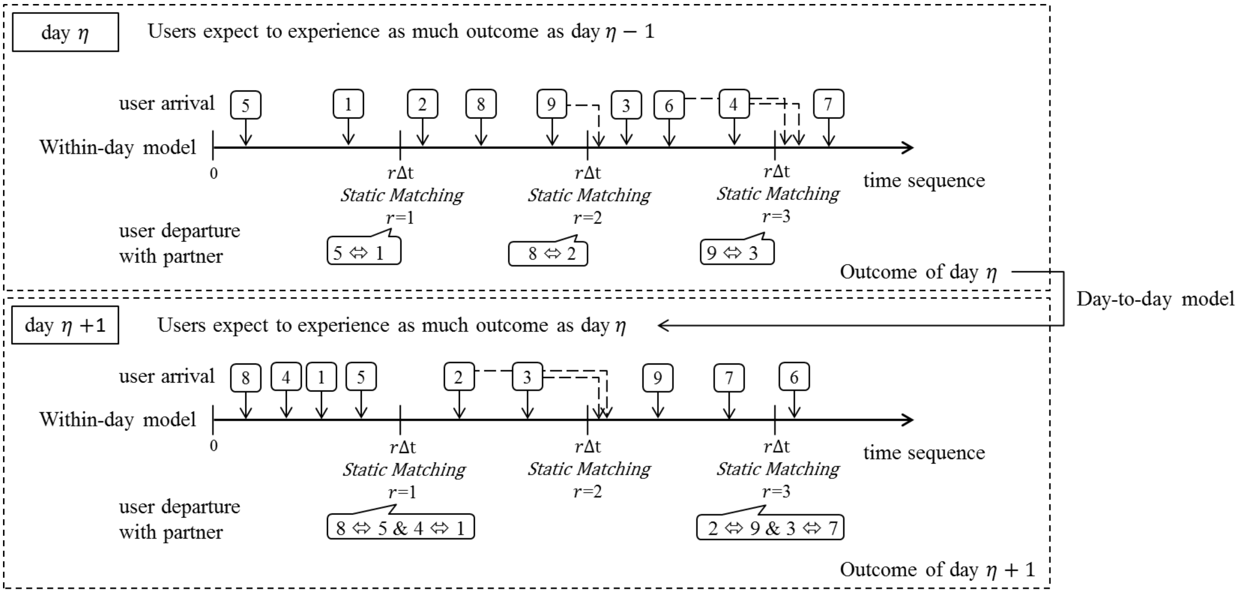


Fig. 2. Conceptual framework of static matching, within-day and day-to-day models for proposed smart ridesharing systems.

The proposed smart ridesharing system is a system where all users can access the system through internet enabled devices or smartphones. Users refer to potential passengers who travel from the same origin (e.g., taxi rank in city center) to different destination (e.g., residence) in two-dimensional city as shown in Fig. 1. In order to focus on ridesharing, the choices of travel mode are limited to two choices: ridesharing and riding alone. For ridesharing, we assume that users share a ride driven by a professional driver (e.g., taxi driver) to only consider passengers' preference. Moreover, we limit number of available passenger seats in one vehicle to two seats to initially study on the simplest problem which is one-on-one passenger matching problem. With these assumptions, riding alone implies that user rides on a vehicle driven by a professional driver alone. In addition, we assume that vehicle supplies at origin are always sufficient in order to neglect the additional waiting time for available vehicle.

Once users access the system, users are required to provide their considered attributes for a preferable ridesharing partner. These attributes can be used to determine user's utility function; however, this preference is formulated as a travel utility in this study which is described in section 3. The static matching is set to be performed at every specific time period $\Delta$t throughout a day as illustrated in Fig. 2. Users who decide to participate in ridesharing system will be involved in the incoming matching. After the matching, users can further decide whether to share a ride with a suggested partner from the system. If user decides to rideshare with suggested partner, user can immediately leave the system and rideshare with the ridesharing partner. However, these two decisions (i.e., participating in ridesharing system to find ridesharing partner, and ridesharing with suggested ridesharing partner) are explained in the within-day model, section 4. For each day, users are informed about previous day's system performance for their decision-making criteria in current day. System performance evaluation for each day and how previous day's system performance can be used are explained in day-to-day model, section 5.

## 3. Static Matching Model

In the proposed SRS, the static matching is repeatedly performed at every $\Delta$t in each day to obtain the matching solution among all users in each matching round $r$ as illustrated in Fig. 2. Users in matching round $r$ include existing users and new users. The existing users in round $r$ are the users who appear in the system before $(r-1)\Delta$t and still cannot be matched with a preferable ridesharing partner. The new users in round $r$ are users who recently appear in the system during $[(r-1)\Delta\text{t}, r\Delta\text{t})$. For instance of matching round $r$ = 3, day $\eta$ in Fig. 2; the static matching is performed on existing user (i.e., user 9) and new users (i.e., user 3, user 4, and user 6).

The one-on-one passenger matching problem developed by Thaithatkul et al. (2015a) is adopted to represent the static matching of each matching round $r$ in proposed SRS. This one-on-one passenger matching problem is modified from a stable roommates problem in (Irving, 1985). The stable roommates problem originally provides a stable matching solution between two different users by considering user preference. This matching solution can represent a travel mode of ridesharing between two travelers. In the modified model, the matching solution between the same user is allowed to consider riding alone solution when there is no preferable ridesharing partner. Therefore, for each matching round $r$, this matching suggests users for a stable matching solution among all users in matching round $r$. Stable matching is a set of pairs where there is no one prefer each other than their partner in current pair. The input of this matching is a preference list of all users in matching round $r$. Each user's preference list contains all users in matching round $r$ (including preference list owner) sorted according to user's preference. However, each user's preference is formulated as travel utility in this study which is described in following subsections. As previously mentioned, the result from stable matching can either be a pair between user $i$ and user $j$ which means that user $j$ is user $i$'s suggested ridesharing partner from the stable matching, or a pair between user $i$ and user $i$ which means that user $i$ has no suggested ridesharing partner under this stable matching. More detail of this matching algorithm can be found in Thaithatkul et al. (2015a).

### *3.1 Assumptions of travel utility*

The modified stable roommates problem matches users based on users preference. The model allows users to freely evaluate and rank others in a list based on their preference. In practice, the preference lists can be obtained with pre-determined utility function instead of a manual evaluation. However, in this study, the preference of user $i$ to user $j$ is formulated as a travel utility $u_{ij}$ where user $i$ and user $j$ are members of users' arrival sequence $S$. In user $i$'s preference list, all users are sorted based on this travel utility. To simplify the computation, the following assumptions are considered.

- Users travel in two-dimensional city with one origin.
- Users' delay cost function is linear over time.
- Travel time is proportional to the Euclidian distance.
- Travel time is fixed as there is no traffic congestion.
- Travel cost refers to a fare which is proportional to the Euclidian distance.
- Users' arrival sequence $S$ is assumed to be an exponential distribution.

The two-dimensional city with one origin means that ridesharing may cost more than riding alone. The delay cost refers to a waiting time of user to be matched with a preferable ridesharing partner which we assume its function to be linear over time for an easy computation. Travel distance is calculated as Euclidian distance and assumed to be proportional to travel time. Moreover, we also assume that travel time is fixed as there is no traffic congestion to make direct travel cost and time constant throughout the matching algorithm. As we consider that a driver in this ridesharing system is a professional driver, user's travel cost then refers to a fare which is also assumed to be proportional to the Euclidian distance. Lastly, we assume that users' arrival sequence $S$ is an exponential distribution.

### *3.2 Formulation of travel utility*

Based on the above assumptions, user preference can be formulated as travel utility $u_{ij}$ as follows.

$$u_{ij}(\tau_i) = \alpha_1 x_{ij} d(\mathbf{a},\mathbf{o}) + \alpha_2 \tau_i \qquad \text{for } i,j \in S \tag{1}$$

$$x_{ij} d(\mathbf{a},\mathbf{o}) = \begin{cases} 0 & \text{for } i,j \in S, i = j & \text{(2.a)} \\ d(\mathbf{a},\mathbf{o}) - \left[\frac{d(\mathbf{b},\mathbf{o})}{2} + d(\mathbf{a},\mathbf{b})\right] & \text{for } i,j \in S, i \neq j, d(\mathbf{a},\mathbf{o}) > d(\mathbf{b},\mathbf{o}) & \text{(2.b)} \\ d(\mathbf{a},\mathbf{o})/2 & \text{for } i,j \in S, i \neq j, d(\mathbf{a},\mathbf{o}) \leq d(\mathbf{b},\mathbf{o}) & \text{(2.c)} \end{cases}$$

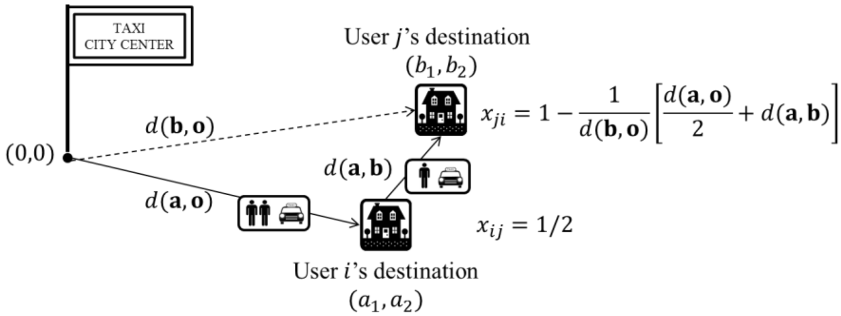


Fig. 3. Example of percentage of cost reduction.

where,

- $u_{ij}(\tau_i)$ : travel utility of user $i$ when matches with user $j$, where $i = j$ means riding alone and $i \neq j$ means ridesharing,
- $\tau_i$ : waiting time of user $i$,
- $x_{ij}$ : percentage of cost reduction for user $i$ traveling with user $j$,
- $\alpha_1$ : marginal utility for saving one unit of money, $\alpha_1 > 0$,
- $\alpha_2$ : marginal disutility for waiting one unit of time, $\alpha_2 < 0$,
- $d(\mathbf{a},\mathbf{b})$ : Euclidian distance between point **a** and **b**,
- **a** : Cartesian coordinates of user $i$'s destination,
- **b** : Cartesian coordinates of user $j$'s destination,
- **o** : Cartesian coordinates of origin, which is (0, 0).

In Equation (1) for $i \neq j$, the first term represents a utility of travel cost reduction when user $i$ rideshares with user $j$, while the second term represents a disutility of user $i$'s waiting time for user $j$. In case $i = j$, this equation represents a travel utility of riding alone. The travel utility matrix is asymmetric where $u_{ij} \neq u_{ji}$, unless travel cost and arrival time of user $i$ and user $j$ are equivalent. As we assume that direct travel time and cost are constant throughout the matching algorithm, a travel cost is simply represented by distance.

According to our first assumption, the utility of travel cost reduction can possibly be negative as ridesharing may cost more than riding alone because of detour. The cost of riding alone for user $i$ is represented by a Euclidian distance from origin to user $i$'s destination denoted as $d(\mathbf{a},\mathbf{o})$. The percentage of cost reduction for ridesharing comparing to riding alone $x_{ij}$ can be calculated through a cost reduction function in Equation (2) and shown in Fig. 3. In case $i = j$, the $x_{ij}$ is always equal to zero as riding alone cannot reduce any travel cost as shown in Equation (2.a). For $i \neq j$, a closer destination is visited first in order to minimize the total travel distance from origin to the last destination. Hence, the distance from origin to the first destination is shared by two users, while the distance from the first destination to the second destination will not be shared by another user. Therefore, if user $i$'s destination is farther from origin comparing to user $j$'s destination, user $i$'s cost reduction and percentage of cost reduction $x_{ij}$ can be calculated by using Equation (2.b). In this case, if the detour from origin to user $j$'s destination is too long, $x_{ij}$ can be negative implying that ridesharing with user $j$ costs more than riding alone. If user $i$'s destination is closer, then the user $i$'s travel cost is shared by user $j$ as shown in Equation (2.c), so the percentage of cost reduction is always at 0.5.

## 4. Within-day Model

The static matching provides users a suggested ridesharing partner from all users in each matching round. In this formulated within-day model, if users are not satisfied with their suggested ridesharing partner, users are allowed to wait for the next matching round. The assumptions and details of formulated within-day model are described in this section.

### 4.1 Assumptions

The process of within-day model and its formulation are developed based on following assumptions.

- Users' arrival sequence $S$ is assumed to be an exponential distribution.
- Users have an expectation on outcome from SRS.
- Users' expectation on outcome from SRS is obtained from day-to-day model.
- Users make two decisions in within-day model: whether to participate in SRS, and whether to rideshare with suggested ridesharing partner.
- The static matching is adopted at every specific time, $\Delta t$.
- The static matching is represented by a modified stable roommates problem.

To describe the arrival time of individual user that randomly arrives at the origin, users' arrival sequence $S$ is assumed to be an exponential distribution. Users make their decisions in within-day model based on their expectation on outcome from SRS. Since this expectation cannot be determined through within-day model, we therefore assume to obtain the expectation from day-to-day model which is described in section 5. In within-day model, there are two decisions which user has to make. The first decision is once user arrives at origin; user has to decide whether to participate in SRS. If user decides to participate in SRS, user is involved in the incoming static matching. The second decision is after static matching suggests user a ridesharing partner; user has to decide whether to rideshare with that suggested ridesharing partner. If user decides to rideshare with the suggested ridesharing partner, user can immediately leave the system and make a trip with that partner (e.g., user 2 and user 8 in matching round $r = 2$ day $\eta$, Fig. 2). In case, user decides not to rideshare with the suggested ridesharing partner, user is allowed to wait for the next static matching round (e.g., user 9 in matching round $r = 2$ day $\eta$, Fig. 2). The static matching is performed at every specific time $\Delta t$ in order to provide users a suggested ridesharing partner among all users in each matching round $r$ as illustrated in Fig. 2. The static matching is represented by a modified stable roommates problem which is described in previous section.

### 4.2 Formulation

Users' expectation on participating in SRS is an expected travel utility. Equation (3) represents user $i$'s expected travel utility on SRS denoted as $EU_i^{RS}(\tau_i|S,\mathcal{A})$, which is written based on Equation (1). This is a function of user $i$'s waiting time for a preferable ridesharing partner (user $j$) which is dependent on arrival sequence $S$ and static matching algorithm $\mathcal{A}$. In the same way as Equation (1), it consists of expected cost reduction and expected waiting time where $EX_i$ and $EW_i(\tau_i)$ denote user $i$'s expected percentage of cost reduction in ridesharing and waiting time for preferable partner, respectively. As previously mentioned that this expectation (i.e., $EX_i$ and $EW_i$) cannot be determined through within-day model, we therefore develop a day-to-day model to consider previous experience as today's expectation as illustrated in Fig. 4 which is further described in section 5.

$$EU_i^{RS}(\tau_i|S,\mathcal{A}) = \alpha_1 EX_i d(\mathbf{a},\mathbf{o}) + \alpha_2 EW_i(\tau_i) \tag{3}$$

Fig. 4 illustrates an integrated model of the proposed SRS. The process of within-day model is shown in dashed line box, while outside dashed line box area shows day-to-day model. In within-day model, there are two decisions that each user has to make. Once user $i$ arrives, user has to decide whether to participate in SRS or not as shown in the most left diamond box in Fig. 4. User will participate in SRS and be involved in the incoming static matching round $r$ only if ridesharing is expected to be worth than riding alone; or Equation (4) is satisfied. $EU_i^A(\tau_i)$ denotes an expected travel utility of riding alone. In case that ridesharing is not expected to be worth than riding alone; or Equation (4) is not satisfied, user will not participate in SRS and immediately travel alone to receive the maximum $EU_i^A(\tau_i)$ at zero. Since both $EU_i^{RS}(\tau_i|S,\mathcal{A})$ and $EU_i^A(\tau_i)$ are linearly reduced overtime, if ridesharing is expected to be better than riding alone at this stage, riding alone will never become better than ridesharing which implies that

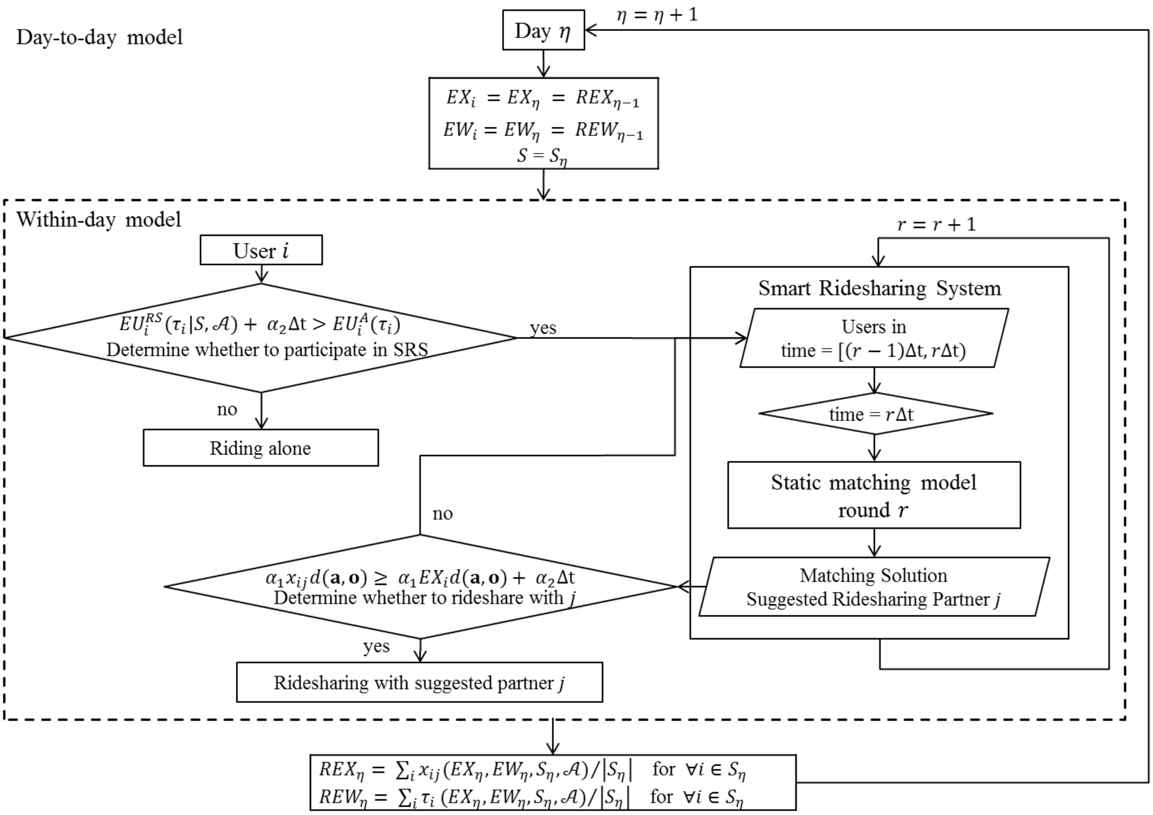


Fig. 4. Integrated matching model for passenger matching problem in smart ridesharing systems.

once user decides to participate in SRS for finding ridesharing partner, user will never leave the system without partner.

$$EU_i^{RS}(\tau_i|S,\mathcal{A}) + \alpha_2\Delta t > EU_i^A(\tau_i) \tag{4}$$

The static matching round $r$ is performed at $r\Delta t$ on new users whose Equation (4) is satisfied and existing users. From the result of static matching, each user is informed the suggested ridesharing partner among users in that matching round. Then, user has to make second decision whether to rideshare with that suggested ridesharing partner. User will rideshare with that partner only if his current partner can provide a cost reduction greater than or equal to what he expects to receive in next matching round; or Equation (5) (i.e., bottom diamond box in Fig. 4) is satisfied. Otherwise, user does not have to leave the system and wait for a preferable ridesharing partner in the next matching round. These users, who wait for the next matching round, are called existing users. And this within-day matching process is repeated throughout a day.

$$\alpha_1 x_{ij} d(\mathbf{a},\mathbf{o}) \geq \alpha_1 EX_i d(\mathbf{a},\mathbf{o}) + \alpha_2\Delta t \tag{5}$$

## 5. Day-to-day Model

In within-day model, users have to make decisions based on their expectation on SRS. However, this expectation cannot be determined in within-day model. We therefore develop the day-to-day model in this section. This day-to-day model allows users to consider the previous SRS performance as information for decision-making criteria.

*5.1 Assumptions*

In order to keep the simplicity of the model, we therefore develop the day-to-day model based on following assumptions.

- Users' arrival sequence $S_\eta$ can be different for different day $\eta$.
- Simple information updating method is considered for day-to-day model.
- All users are informed the same average SRS performance of previous day before they participate in SRS.

In different day, users may arrive with different arrival sequence $S_\eta$ and experience different outcome (e.g., travel mode, departure time, ridesharing partner) as illustrated in Fig. 2. The SRS performance of previous day from day-to-day model is used as information for users to make decisions in within-day model. In this research, a simple updating method is considered as follows. For each day, the SRS performance is evaluated by averaging percentage of cost reduction and waiting time. In the following day, users are informed about the SRS performance of previous day through any communication method. Users therefore make decisions in within-day model based on this information. In other words, users' expectation on current day's SRS performance is set at least equal to previous day's SRS performance.

*5.1 Formulation*

The SRS performance of day $\eta$ can be evaluated in term of revealed expected percentage cost reduction and waiting time, denoted as $REX_\eta$ and $REW_\eta$, respectively. These $REX_\eta$ and $REW_\eta$ can be calculated by averaging $x_{ij}$ and $\tau_i$ of all users in day $\eta$ as shown in Equations (6) and (7), respectively. As users' arrival sequence ($S_\eta$) and expectation on SRS performance ($EX_\eta$ and $EW_\eta$) can be different among days, the outcome of SRS from each day (i.e., $REX_\eta$ and $REW_\eta$) can be different.

$$REX_\eta = \sum_i x_{ij}(EX_\eta, EW_\eta, S_\eta, \mathcal{A})/|S_\eta| \quad \text{for } \forall i \in S_\eta \tag{6}$$

$$REW_\eta = \sum_i \tau_i\,(EX_\eta, EW_\eta, S_\eta, \mathcal{A})/|S_\eta| \quad \text{for } \forall i \in S_\eta \tag{7}$$

The simple updating method of day-to-day model is explained as follows. Once users arrive at the origin or appear in the system, all users are informed the same previous day's SRS performance (i.e., $REX_{\eta-1}$ and $REW_{\eta-1}$) through any communication method. Thus, all users expect to receive the same outcome from current day's SRS at $EX_\eta$ and $EW_\eta$, which is as much as previous day's outcome as shown in Equations (8) and (9) and Fig. 4.

$$EX_i = EX_\eta = REX_{\eta-1} \qquad \text{for } \forall i \in S_\eta \tag{8}$$

$$EW_i = EW_\eta = REW_{\eta-1} \qquad \text{for } \forall i \in S_\eta \tag{9}$$

With the information of $REX_{\eta-1}$ and $REW_{\eta-1}$ and above updating method, user $i$'s expected travel utility for ridesharing in Equation (3) can be rewritten as shown in Equation (10) for all users in day $\eta$.

$$EU_i^{RS} = \alpha_1 EX_\eta d(\mathbf{a}, \mathbf{o}) + \alpha_2 EW_\eta \qquad \text{for } i \in S_\eta \tag{10}$$

With this updating method, the day-to-day equilibrium of SRS can be realized when the revealed outcome is the same as what it's expected, i.e., $REX_\eta = EX_\eta, REW_\eta = EW_\eta$ as represented in Fig. 5 which has already been described in Thaithatkul et al., (2015b). The characteristics of the day-to-day equilibria (i.e., existence, uniqueness, and stability) can also be understood. For instance, the equilibrium is stable if the revealed outcome always converges to from any initial perceived expected outcome. On the other hand, the equilibrium is unstable when the revealed outcome is always diverges from.

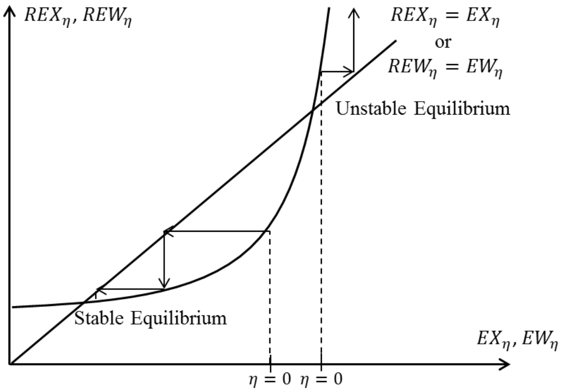


Fig. 5. Conceptual figure of day-to-day equilibria in smart ridesharing systems.

## 6. Integrated Model

The model of our proposed SRS is an integrated model of static model, within-day model and day-to-day model as illustrated in Fig. 4. Users will make their decisions in within-day model based on information from day-to-day model. The static matching model is used to provide a suggested ridesharing partner to users.

For instance, when user arrives at the origin, user will be informed about the SRS performance (i.e., average percentage of cost reduction and average waiting time which are obtained from day-to-day model) of previous day. Then, user has to decide whether to participate in SRS in within-day model. Based on previous day SRS performance, if participating in SRS is expected to be worth than riding alone based on previous SRS performance, user will participate in SRS. Once user decides to participate in SRS, this user will be involved in the incoming static matching model. This static matching model provides a suggested ridesharing partner from all users who exist in the system during that round. The static matching is performed every $\Delta t$. After that, user has to make the second decision whether to rideshare with this suggested ridesharing partner in within-day model. Based on previous day SRS performance again, if this suggested ridesharing partner is expected to be better than waiting for another partner in next matching round, user will leave the system by ridesharing with the suggested ridesharing partner. Otherwise, user will wait in the system for the next matching round.

## 7. Simulation

As previously mentioned, the day-to-day equilibria and their characteristics in SRS are essential, but they are difficult to be investigated by analytical analysis. Hence, the simulation is conducted and explained in this section.

### *7.1 Simulation setting*

To simulate the day-to-day model, we conduct the simulation by giving all possible pairs of $EX$ and $EW$ to represent the possible expected outcome of day $\eta$. The possible $EX$ is set to be a value between $(0,0.5]$ as users always expect to reduce some of their travel cost when rideshare and they can reduce at the maximum of 50% when they rideshare with only one another. The possible $EW$ is given as an integer from 0 until a number that there is no user who chooses to participate in SRS. The Cartesian coordinate of user's destination in 2-dimensional (x-y) city is given by a random number from the multivariate normal distribution. We consider two cases of the distribution of user's destination as follows.

- Case 1: Multivariate normal distribution with mean at 0 and 0, and covariance at [1089, 0] and [0, 1089] for dimensions x and y, respectively ($\mathbf{a} \sim N([0,0],[1089,0;0,1089])$).
- Case 2: Multivariate normal distribution with mean at 50 and 0, and covariance at [1089, 0] and [0, 1089] for dimensions x and y, respectively ($\mathbf{a} \sim N([50,0],[1089,0;0,1089])$).

The parameters $\alpha_1$ and $\alpha_2$ in travel utility function were given equal to 1 and $-1$, respectively. Users' arrival sequence, $S$, has an exponential distribution with parameter $\lambda_S$ equal to 1 user/minute. With this setting, we can obtain a real outcome of percentage of cost reduction and waiting time for a partner that users experience when they expect at $EX$ and $EW$. By averaging the outcomes of each day as Equations (6) and (7), the $REX$ and $REW$ for given $EX$ and $EW$ can be obtained.

### *7.2 Results of simulations*

We simulated 200 matching rounds to represent one day. The following results were obtained during 51st round to 150th round to represent the results during the day. For each possible pair of $EX$ and $EW$, we performed 20 times to obtain the average result.

Fig. 7 and Fig. 8 represent a vector field of day-to-day dynamics of one-on-one passenger matching problem under two-dimensional city for case 1 and case 2, respectively. In these figures, the pairs of $EX$ and $EW$ which do not have an arrow mean that no user is satisfied the first or second decisions in Equations (4) and (5), respectively. In other words, ridesharing is not expected to be worth than riding alone; or user is not satisfied with suggested ridesharing partner. These areas without arrow (i.e., north-western areas) represent an existence of undesirable equilibrium for both cases where no one participates in SRS. In other words, SRS is not feasible for these pairs of $EX$ and $EW$.

For the pairs of $EX$ and $EW$ which have arrow, the head of an arrow shows the direction to $REX$ and $REW$ where black arrow, blue arrow, green arrow and red arrow represent the direction to southwest, southeast, northeast and northwest, respectively. The length of an arrow shows a proportional distance to $REX$ and $REW$. For instance in Fig. 7, if users are informed that $EX = 10$ and $EW = 5$ in a certain day, the $REX$ and $REW$ of this day will be 29 and 5, respectively. Then, the $EX$ and $EW$ of the following day are 29 and 5, respectively. However, the simulation has a limitation to define equilibrium, as we might not be able to obtain the point where revealed outcome is exactly the same as expected outcome as illustrated in Fig. 5. By using vector field, the stable equilibrium is therefore defined by the point where the arrows point to as illustrated as an example in Fig. 6.

For case 1, the direction of arrows in Fig. 7 point to the circled area which is outside the feasible area of $EX$ and $EW$. This means that the day-to-day equilibrium where SRS is feasible does not exist under case 1's conditions as the revealed expected waiting time is not worth for users to participate in SRS. The arrows which point to the different direction from other arrows (around the boundary of ridesharing and riding alone in bottom left area of Fig. 7) are caused by the insufficient number of users whose Equations (4) and/or (5) are not satisfied during the 51st round to 150th round. For case 2, the existence of a stable day-to-day equilibrium where SRS is feasible can be realized within the circled area in Fig. 8 which is around the same area that arrows in Fig. 7 point to. Besides that, Fig. 8 also shows that users with longer travel distance can join SRS with longer expected waiting time. By comparing the results between these two cases, it shows that SRS might not be feasible for a group of users with short travel distance and different direction of destinations as there is no existence of stable equilibrium under conditions in case 1. On the other hand, SRS is feasible for a group of users with longer travel distance and same direction of destinations.

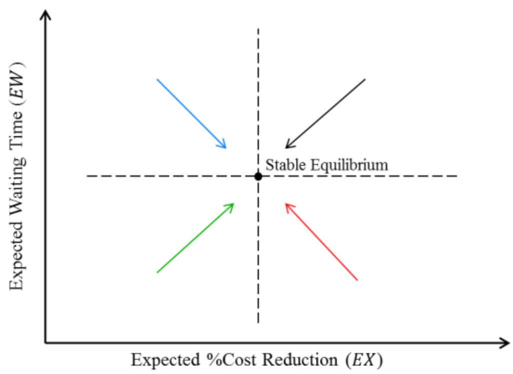


Fig. 6 Example of stable equilibrium from simulation result.

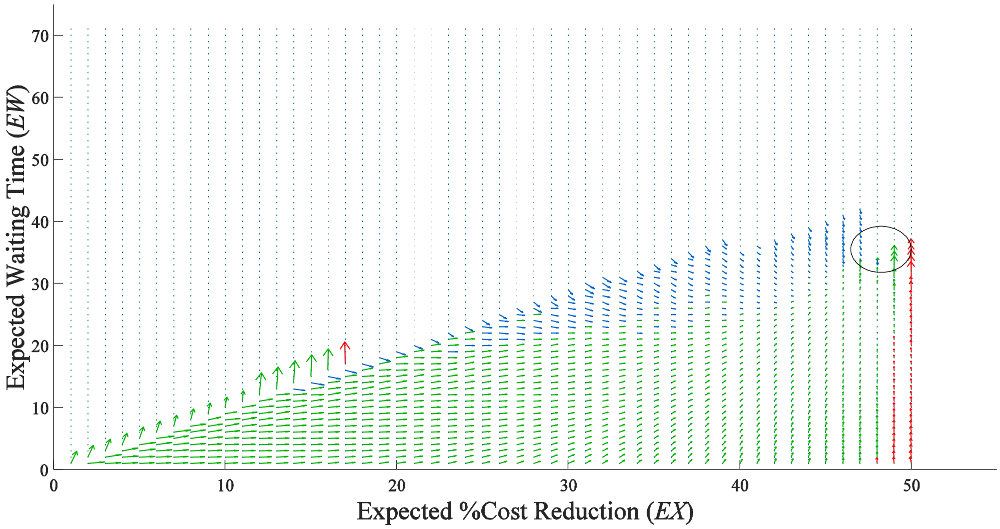


Fig. 7. Day-to-day dynamics of passenger matching problem under two-dimensional city where $\mathbf{a} \sim N([0,0],[1089,0;0,1089])$.

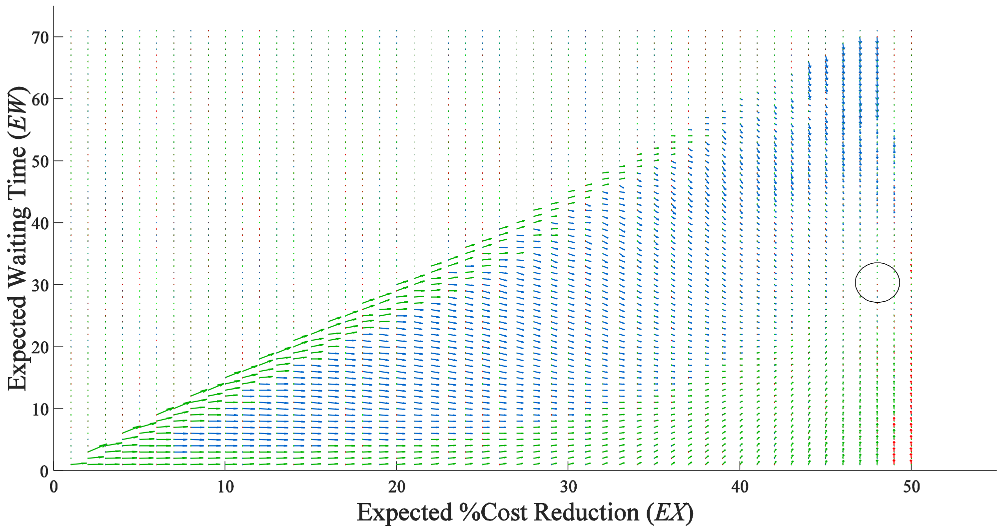


Fig. 8. Day-to-day dynamics of passenger matching problem under two-dimensional city where $\mathbf{a} \sim N([50,0],[1089,0;0,1089])$.

By comparing the results with the day-to-day dynamics of SRS under a simple condition of one-dimensional city in Thaithatkul et al., (2015b), we can realize that the revealed waiting times in stable equilibria for both cases of two-dimensional city (this study) are significantly longer than the revealed waiting time of one-dimensional city. This is caused by the possibility that ridesharing with someone may cost more than riding alone, so it becomes more difficult to find a preferable ridesharing partner. In term of revealed percentage of cost reduction, both cases of two-dimensional city provide only a slightly lower percentage of cost reduction comparing to one-dimensional city.

To investigate the characteristics of matched ridesharing partner, we therefore find the relationship between user's travel distance from origin and detour distance caused by matched ridesharing partner. We investigate this characteristic through the simulation as it is difficult to analytically investigate as matched ridesharing partner can only be determined through the model which is dependent on $EX$, $EW$, $S$ and $\mathcal{A}$. From the result of the simulations, detour distance caused by matched ridesharing partner varies according to the distance of user's destination from origin as illustrated in Fig. 9 and Fig. 10 for case 1 and case 2, respectively. These two cases result in similar pattern of logarithm relationship. The dramatically increasing for users whose destination is very far can be caused by the

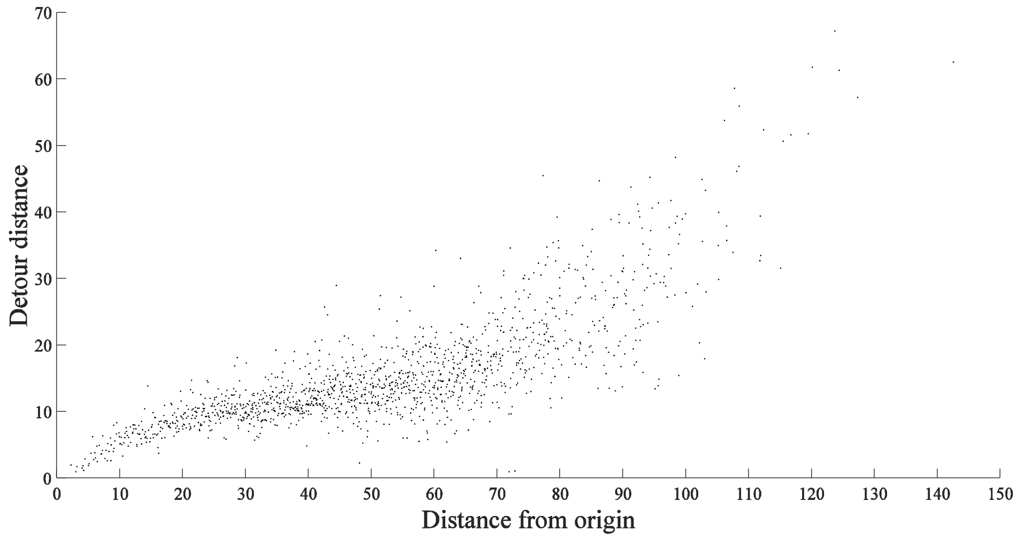


Fig. 9. Relationship between distance from origin and detour distance under two-dimensional city where $\mathbf{a} \sim N([0,0],[1089,0;0,1089])$.

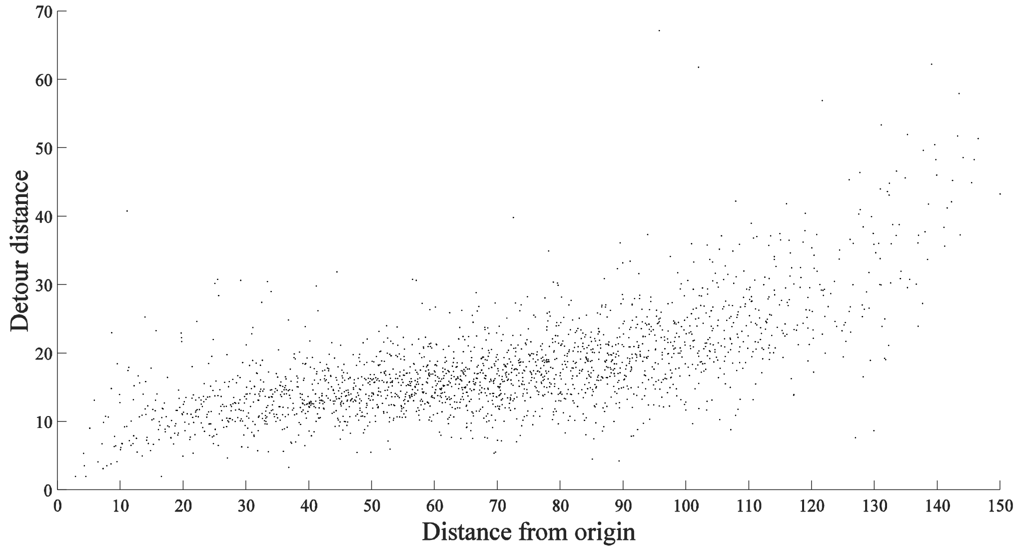


Fig. 10. Relationship between distance from origin and detour distance under two-dimensional city where $\mathbf{a} \sim N([50,0],[1089,0;0,1089])$.

decreasing number of users. However, this can be determined by the distribution of users' destination. In addition, these results can also imply to acceptable detour distance of each user to rideshare. By knowing the characteristic of matched ridesharing partner, we can arrange an appropriate group of users based on users' destination to perform the matching. Since the matching is performed on a group of lower number of users, the time consuming of matching process can be reduced.

## 8. Conclusion

This study aims to investigate the day-to-day dynamics of one-on-one passenger matching problem in smart ridesharing systems (SRS) under general assumption of two-dimensional city. The modified stable roommates problem is adopted to obtain a stable matching of one-on-one passenger matching. This matching method enables SRS to consider user preference in the matching. User preference in this study is formulated as a travel utility which consists of cost reduction and waiting time for ridesharing. Based on this matching method, the matching processes of within-day and day-to-day models are formulated. To investigate the day-to-day dynamics, we perform a simulation of the formulated models under specific conditions.

Through the simulations, the existence of multiple day-to-day equilibria is investigated under some conditions. One of the existing equilibria is an undesirable equilibrium where SRS is not feasible to be operated, while the existence of a desirable stable equilibrium, where SRS is feasible to be operated, is realized under some conditions. The SRS becomes feasible to be operated when SRS can provide users the outcome as much as users expect. The outcome of SRS is evaluated by a percentage of cost reduction for ridesharing and waiting time for a ridesharing partner. At the stable day-to-day equilibrium, an average percentage of cost reduction is quite high at above 45%, while the maximum percentage of cost reduction is at 50% for one-on-one passenger matching problem under the assumed conditions. An average waiting time to be matched with ridesharing partner is a little high, but this waiting time can be reduced with increasing number of users. With the investigation of multiple equilibria of SRS, the SRS service providers can efficiently operate and manage the system to avoid the undesirable equilibrium and lead the system to the desirable one.

Besides that, the characteristic of a matched ridesharing partner is also realized as a relationship between user's travel distance from origin and detour distance caused by matched ridesharing partner. From the simulation, user's travel distance and detour distance have logarithm relationship. By knowing the characteristic of matched ridesharing partner or acceptable detour distance of each user, we can arrange an appropriate group of users based on users' destination to perform the matching which can also reduce the time consuming in matching process of SRS.

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